More on the derivative

1. Suppose that f is differentiable at x. Show that the limit

$$\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}$$

exists and is equal to $f'(x)$. Show that if this limit exist then f is not necessarily differentiable.

2. Find $f'(0)$ if

$$f(x)=\begin{cases} g(x)\sin(1/x), & x\neq 0\\ 0, & x=0\end{cases}$$

and $g(0)=g'(0)=0$.

Computing derivatives

3. If f is differentiable at a, let $d(x)=f(x)-f'(a)(x-a)-f(a)$. Find $d'(a)$.

4 (Logarithmic differentiation)**.** Differentiate

$$f(x)=\sqrt[x]{x},\quad g(x)=\sqrt{\frac{x(x-1)}{x-2}},\quad h(x)=(\cos x)^{\sin x}$$

5. Show that the function $f(x)=xe^{-x^2/2}$ satisfies the differential equation $xf'(x)=(1-x^2)f(x)$

6. The radius of a sphere is increasing at a uniform rate of 5 cm/sec. At what rate are the area of the surface of the sphere and the volume of the sphere increasing when the radius becomes 50 cm.

7. At what point of the curve $y^2=2x^3$ is the tangent perpendicular to the straight line $4x-3y+2=0$?

Review of limits

8. A function is defined by the formulas

$$f(x)=\begin{cases} x\sin(1/x), & x\neq 0\\ A, & x=0\end{cases}$$

how should the value of A be chosen so that f is continuous at 0?

9. Find the limit

$$\lim_{x\to 0}\left(\frac{2+x}{3-x}\right)^x,\quad \lim_{x\to 1}\left(\frac{x-1}{x^2-1}\right)^{x-1}$$

10. Prove the following limit exists

$$\lim_{x\to 5}x^2-x-6=0$$

using the ε-δ definition.